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B.M.S COLLEGE FOR WOMEN

BENGALURU – 560004

I SEMESTER END EXAMINATION – APRIL 2024

M.Sc – MATHEMATICS- DISCRETE MATHEMATICS

(CBCS Scheme-F+R)

Course Code MM105T **QP Code: 11005**

Duration: 3 Hours **Max. Marks: 70**

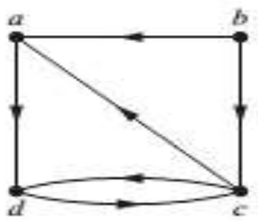
Instructions: ***1) All questions carry equal marks.***
2) Answer any five full questions.

1. a) Obtain the principal conjunctive normal form of $(\sim p \to r) \wedge (q \leftrightarrow p)$ without using truth table.
   b) Give (i) direct proof (ii) indirect proof and (iii) proof by contradiction for the following statement: "If n is an odd integer then $(n + 9)$ is an even integer".
   c) Test the validity of the following argument.
   "If I study then I will not fail in mathematics. If I do not play basketball then I will study. But I failed in mathematics. Therefore, I must have played basketball".
   (4+6+4)

2. a) An urn contains 15 balls, 8 of which are red and 7 are black. In how many ways can we choose
   (i) 5 red balls?
   (ii) 7 balls such that atleast five are red?
   b) Show that if any five numbers are chosen from 1 to 8, then two of them will have their sum equal to 9.
   c) In how many ways can eight identical cookies be distributed among three distinct children if each child receives atleast two cookies but not more than four cookies?
   (4+5+5)

3. a) Model the rabbit population as recurrence relation and solve it explicitly.
   b) Using generating functions solve the recurrence relation $a_n = 6a_{n-1} - 8a_{n-2} + 3^n$ $(n \geq 2)$ with initial conditions $a_0 = 3$ and $a_1 = 7$.
   c) Solve the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2}$ with $a_0 = 1$ and $a_1 = 6$.
   (5+5+4)

4. a) Using Warshall's algorithm find the transitive closure of the relation R on $A = \{a, b, c, d\}$ given by



b) If $(L, \preccurlyeq)$ is a lattice then prove the following:

(i) $a \preccurlyeq b \Longrightarrow a \vee c \preccurlyeq b \vee c$.

(ii) $a \preccurlyeq b \Longrightarrow a \wedge c \preccurlyeq b \wedge c$.

(iii) $a \preccurlyeq b \text{ and } c \preccurlyeq d \Longrightarrow a \vee c \preccurlyeq b \vee d$.

(iv) $a \preccurlyeq b \text{ and } c \preccurlyeq d \Longrightarrow a \wedge c \preccurlyeq b \wedge d$ where $a, b, c, d \in L$.

c) Draw the Hasse diagram representing the partial ordering $\{(a, b) \mid a \text{ divides } b\}$ on $\{1, 2, 3, 4, 6, 8, 12, 24, 36\}$.

(5+5+4)

5. a) State and prove the Handshaking lemma. Hence prove that the number of odd degree vertices in a graph is always even.

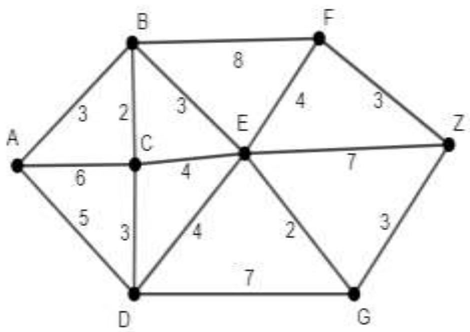
b) Define a self – complementary graph $\overline{G}$ of a graph G and give an example. Show that every self – complementary graph has $4n$ or $4n + 1$ vertices, where n is a positive integer.

c) Define eccentricity, radius $r(G)$, center and diameter $d(G)$ of a graph G. Prove that for any connected graph G, $r(G) \leq d(G) \leq 2r(G)$.

(5+5+4)

6. a) Prove that in a graph G, $k(G) \leq \lambda(G) \leq \delta(G)$, with standard notations.

b) Using Dijkstra's algorithm find the shortest path from 'a' to all other vertices, in the following graph.



c) Prove that an edge e in a graph G is a bridge if and only if e does not lie on any cycle of G.

(5+5+4)

7. a) Prove that a non-trivial connected graph G is Eulerian if and only if degree of every vertex of G is even.

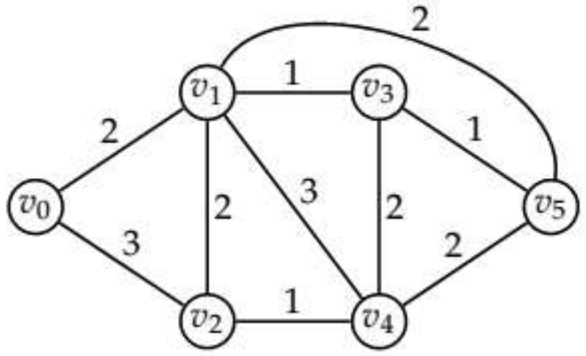
b) State and prove Ore's theorem for a Hamiltonian graph.

c) State and prove Euler's polyhedron formula.

(5+5+4)

8. a) Prove that every tree has one or two central vertices.

   b) For the following weighted graph find the minimum spanning tree using Kruskal's algorithm.



   c) Define a binary tree. Prove that the number of pendent vertices in a binary tree with p vertices is $\left(\frac{p+1}{2}\right)$.

(5+5+4)

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